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Abstract

The theory of domination is one of the most studied fields in graph theory. Many new domination parameters have been defined and studied so far. One such parameter that was introduced in 2017 is partial domination number. For a graph $G = (V,E)$ and for a $p \in (0,1]$, a subset S of $V(G)$ is said to partially dominate or p-dominate G if $|N[S]| \geq p|V(G)|$. The cardinality of a smallest p-dominating set is called the p-domination number and it is denoted by $\gamma_p(G)$. In scenarios wherein domination concepts are applied, partial domination concepts can also be applied with the added advantage of being able to dominate the underlying graph partially, when the need arises. This advantage makes this parameter appear unique amongst most other domination parameters. Also, very little work has been done in this area. Hence we felt motivated enough to do research on this parameter and in this thesis we study partial domination, which in a sense generalizes the concept of classical domination. We present some basic properties of partial dominating sets, some properties related to particular values of 'p', some properties related to the eccentricity of a p-dominating set, some results in the line of classical domination and characterizations of a p-dominating set. Then we study partial domination in the context of prisms of graphs with respect to a permutation π of $V(G)$. We give some some bounds for partial domination numbers of prisms of graphs G in terms of partial domination numbers of G for particular values of 'p'. We define universal γ_p-fixers and universal γ_p-doublers and we characterize paths, cycles and complete bipartite graphs which are universal $\gamma_{\frac{1}{2}}$ - fixers and universal $\gamma_{\frac{1}{2}}$ - doublers. Then we concentrate on establishing a domination chain in the context of partial domination. For this, we defined independent partial domination number(IPD-number), found exact values of IPD-numbers for some classes of graphs, found bounds for IPD-numbers in terms of independent domination number and some relations between the independent partial dominating sets and the independent dominating sets. Then we defined partial independence number, partial irredundance number and proved some results that lead us toward the partial domination chain. We also present some relations that exist amongst the parameters in the partial domination chain. We give a definition for p-well covered graphs and study them for some classes of graphs.

Keywords: domination, partial domination, universal γ_p-fixers and γ_p - doublers, independent partial domination number, partial independence, partial irredundance, partial domination chain

Contents

List of Figures **v**

1 Introduction **1**
1.1 Statement of the research problem and objectives 2

2 Review of Literature **3**
2.1 History of the theory of domination 3
2.2 Partial domination 3

3 Some Properties of Partial Dominating Sets **5**
3.1 Basic properties 5
3.2 Properties of p-dominating sets for particular values of p 6
3.3 Properties related to eccentricity of a p-dominating set 6
3.4 Properties in the line of classical domination 7

4 Partial Domination in Prisms of Graphs **8**
4.1 Relations between p-domination number of a graph and that of its prisms 8
4.2 Universal γ_p-fixers and universal γ_p-doublers 10

5 Partial Domination Chain **11**
5.1 Independent partial domination 12
5.2 Partial independence number 13
5.3 Partial irredundance number 14
5.4 Relations between the parameters in the partial domination chain . 15
5.5 p-well covered graphs 16

6 References **17**

LIST OF PUBLICATIONS **20**

LIST OF CONFERENCE PRESENTATIONS **20**

List of Figures

1 Partial Domination 2
2 Graph and Prism of Graph 8
3 Graph with $\beta_{\frac{4}{5}} = 2$ 13
4 Graph with $ir_{\frac{4}{5}} = 1$ and $IR_{\frac{4}{5}} = 2$ 15

This thesis deals with the theory of partial domination in graphs, which generalizes the theory of domination in the context of extent of domination. We focus our study on finite, simple and undirected graphs, $G = (V,E)$ with vertex set V and edge set E.

The theory of domination is one of the most studied and active areas in graph theory. Owing to its applications in many fields like computer networks, social networks etc., domination theory has attracted researchers from many disciplines.

1 Introduction

We present our research findings in 5 Chapters. In Chapter 1 we give some basic definitions and results which motivated our research.

A dominating set is defined as follows.

Definition 1.1. *Let $G = (V,E)$ be a graph. A set of vertices S is called a dominating set of G if each vertex not in S is adjacent to atleast one vertex in S. The cardinality of a smallest dominating set denoted by $\gamma(G)$ is called the domination number of G and a dominating set having that cardinality is referred to as a $\gamma(G)-$set.*

Many new domination parameters were defined and are being defined to cater to the needs that arise in real life situations. One such parameter is partial domination. To have a better understanding of its definition, we need to know the following notations. For a vertex $v \in V(G)$, its open neighbourhood is defined by $N(v) = \{u \in V(G)/uv \in E(G)\}$ and its closed neighbourhood by $N[v] = N(v) \cup \{v\}$. For a set $S \subseteq V$, $N(S) = \bigcup_{v \in S} N(v)$ is its open neighbourhood and $N[S] = \bigcup_{v \in S} N[v]$ is its closed neighbourhood. Now we go for the definition of a partial dominating set.

Definition 1.2. *For any graph G = (V, E) and proportion $p \in (0,1]$, a set $S \subseteq V$ is a p-dominating or partial dominating set if $\frac{|N[S]|}{|V|} \geq p$. The p-domination or partial domination number $\gamma_p(G)$ equals the minimum cardinality of a p-dominating set in G.*

In light of the definitions of the neighbourhoods, it is obvious that, for a dominating set S, $N[S] = V$. So a partial dominating set, when compared with a dominating set, dominates a proportion 'p' of the vertex set which is not necessarily the whole set and hence partially dominates G. The set of all white vertices in Figure 1 dominates exactly 4 vertices and hence is a $\frac{4}{9}$-dominating set.

As in the case of most other variants of domination, this parameter too was defined by taking into account a practical situation, that may occur in any distribution system. In any distribution system the main criterion is to deliver the resources at every destination and a dominating set

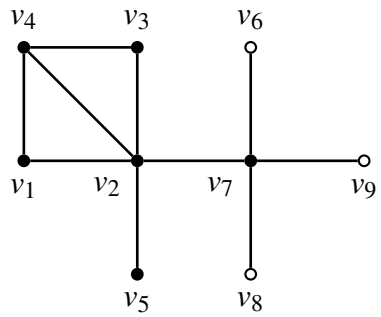


Figure 1: Partial Domination

can aid in this purpose. But in some cases due to limited resources or other reasons, it may not be possible or profitable to deliver at every destination. In such cases, partial domination concepts can come in handy. With the available resources, partial domination techniques can be applied to achieve optimum distribution.

1.1 Statement of the research problem and objectives

The aim of our research was to study partial domination and to get results that lead to generalization of results in classical domination.

Our research had the following objectives and we were able to achieve them.

- To explore the properties of partial dominating sets
- To define and study new partial domination parameters by combining the concept of partial domination with various graph theoretic parameters
- To find the bounds/exact values for partial domination numbers for various families and types of graphs
- To characterize graphs which achieve those bounds

Our research methodology included the investigation of

- Evaluation problems which deal with the evaluation of partial domination numbers
- Existence problems which seek to find graphs with certain given conditions
- Characterization problems which seek to characterize graphs having certain given conditions
- Extension problems which extend partial domination concepts with the help of other parameters

2 Review of Literature

In Chapter 2, we review some of the literature that is available on the theories of domination and partial domination.

2.1 History of the theory of domination

The origin of domination dates back to 1850's when chess enthusiasts studied the problem of determining the minimum number of queens required to cover/dominate all the squares on a 8 x 8 chessboard. Later it was found out that 5 queens are enough for that purpose. In 1862, a person named de Jaenisch tried this problem on an n x n chessboard. In 1958, Claude Berge in his book on graph theory defined the concept of domination for the first time. But he referred to the concept as 'coefficient of external stability'. It was Oystein Ore who first used the terms 'dominating set' and 'domination number'. B. Cockayne and S. Hedetniemi published a survey paper on domination related results in 1977 wherein they first used $\gamma(G)$ to denote the domination number of a graph G. This survey paper became a starting point to an avalanche of massive research in domination. The fact that more than 3000 papers have been published on domination so far, vouch for this. Still now, domination is one of the active research areas in graph theory. The book titled 'Fundamentals of Domination in Graphs', written by T.Haynes et al., [1] provide a comprehensive treatment of domination in graphs followed by [2].

Many varieties of domination have been defined and studied. Suppose $S \subseteq V(G)$ is a dominating set of a graph G. The criteria behind the definitions of some types of domination can be classified based on restrictions on S, restrictions on $V - S$ and restrictions on both S and $V - S$. Some types of domination were defined according to the restrictions based on the weights assigned to each $v \in V$ by means of a function f from V. Some domination parameters were defined in the context of edges and fuzzy graphs and some like domatic number involved more than one vertex set.

2.2 Partial domination

The area of domination is so vast that one cannot cover all the domination parameters that have been studied so far. So we stop here and turn our attention towards our main focus partial domination. Let's see what sets apart the concept of partial domination from all the other domination parameters.

The main thing that is common amongst most of the domination parameters is that, they require that, each vertex in a graph be dominated. Or in other words, a dominating set gets hold of all the vertices in a graph; nothing is left out. But in the case of partial domination, there is a possibility that some vertices may be left out by a partial dominating set; there is

no complete domination. Now, this possibility can have far reaching effects in many real life problems where domination concepts are applied.

Many domination parameters were introduced in response to real life problems that required each vertex to be dominated. But there maybe certain situations wherein complete domination is either non - profitable or impossible. In such cases, partial domination concept can be applied.

In fact, the first study on partial domination was done in response to one such situation. Sensor networks is one area in which domination concept can be applied. In sensor networks, sensors are placed at certain locations to cover the entire monitored area. Minimum number of sensors required for this purpose can be found out using the domination number of the underlying graph.

But sensors in sensor networks are powered by limited-energy batteries and so it is really important to use optimum number of sensors with a required guaranty of coverage, to prolong the network lifetime. It is with this purpose in mind, Liu and Liang [3] studied for the first time in 2005, the partial connected coverage problems in wireless sensor networks. They defined the partial sensor cover as follows:

Definition 2.1. [3] *For a given coverage guarantee of θ with $0 < \theta \leq 1$, the partial sensor cover is a set of sensors with minimum cardinality that covers atleast θ percentage of the entire region.*

They proposed an algorithm related to this and evaluated its performance.

It was in 2017 that, B. M. Case et al. [4] and A. Das [5] first discussed partial domination in the context of general graphs. They formally defined a partial dominating set. Caro and Hansberg [6] were the next to study partial domination. They actually went beyond the concept of partial domination and studied the nature of graphs induced by the non-dominated vertices left out by the partial dominating set. In [6], they concentrate more on the isolation number of a graph. The isolation number of a graph is defined as follows:

Definition 2.2. [6] *Let G=(V, E) be a graph and $\mathscr{F}$ a family of graphs. Suppose $S \subseteq V$, $R(S) = V(G) \backslash N[S]$ is the remainder with respect to S. A set $S \subseteq V$ is $\mathscr{F}$-isolating if the graph induced by the set R(S) contains no member of $\mathscr{F}$ as subgraph. The minimum cardinality of an $\mathscr{F}$-isolating set of a graph G will be denoted as $i(G, \mathscr{F})$ and is called the $\mathscr{F}$-isolation number.*

B. M. Case et al.[7] in 2019, studied some properties of partial dominating sets. In 2019, Macapodi and Isla[8] characterized partial dominating sets of some product graphs and in the same year, they[9] defined total partial dominating sets and studied them in the context of some product graphs.

3 Some Properties of Partial Dominating Sets

In Chapter 3 we present some basic properties of partial dominating sets.

In [4] it was proved that for any graph G, if $p < q$, then $\gamma_p(G) \leq \gamma_q(G)$. We proved the following.

Proposition 3.1. *If in a graph G, $\gamma_p(G) < \gamma_q(G)$, then p<q.*

B. Case et al., in their work[4] concentrated more on $\frac{1}{2}-$domination numbers. They found exact $\gamma_{\frac{1}{2}}-$ values for some classes of graphs and found bounds for $\gamma_{\frac{1}{2}}-$ values for a general graph in terms of its domination number. A. Das[5] in his work concentrated more on γ_p- values for all $p \in (0,1]$. He found bounds for them in terms of the graph theoretic parameters like order of G, maximum degree and domination number. So both in [4] and [5] the focus was more on γ_p-sets. In Chapter 2 we present results mostly related to partial dominating sets in general.

3.1 Basic properties

Proposition 3.2. *Let $p,q \in (0,1]$. If in a graph G, $\gamma_p(G) < \gamma_q(G)$, then $p < q$.*

Proposition 3.3. *Let G = (V, E) be a graph with $H,K \subseteq V$. If K is a minimal q-dominating set and $H \subset K$ is a p-dominating set, then $\gamma_p \leq \gamma_q$.*

Proposition 3.4. *Let H and K be minimal p and minimal q dominating sets respectively. Then $H-K$ is a minimal p-dominating set iff $H \cap K = \phi$.*

Proposition 3.5. *Let G = (V, E) be a graph. Suppose $H,K \subseteq V$ are minimal p and minimal q dominating sets respectively such that $K \subset H$. Then $H-K$ is a $p-\frac{|N[K]|}{|V|}$ dominating set and if $q = \frac{|N[K]|}{|V|}$ then $H-K$ is a $p-q$-dominating set.*

Proposition 3.6. *Let G = (V, E) be a graph. Suppose $H,K \subseteq V$ are minimal p and minimal q dominating sets respectively. Then $H \cup K$ is a $p+q-\frac{|N[H] \cap N[K]|}{|V|}$-dominating set.*

Corollary 3.1. *Suppose $H,K \subseteq V$ are minimal p and minimal q dominating sets respectively such that $N[H] \cap N[K] = \phi$. Then $H \cup K$ is a $p+q$-dominating set.*

Corollary 3.2. *Suppose $H,K \subseteq V$ are γ_p and γ_q sets respectively such that $N[H] \cap N[K] = \phi$. Then $\gamma_{p+q}(G) \leq \gamma_p(G) + \gamma_q(G)$.*

Theorem 3.1. *Suppose $H,K \subseteq V$ are γ_p and γ_q sets respectively such that $N[H] \cap N[K] = \phi$ for $p,q \in (0,1]$. Then $\gamma_{p+q}(G) < \gamma_p(G) + \gamma_q(G)$ iff (i)$\exists \gamma_{p+q}(G)$-set A such that $A \neq B = H \cup K$ and (ii)$\exists S \subseteq A$ and $T \subseteq B$ such that $|S| < |T|$ and $|N[S]| \geq |N[T]|$.*

3.2 Properties of p-dominating sets for particular values of p

In [5] it was proved that γ_p- values vary depending on 'p'-values and that too when $p = \frac{|N[S]|}{|V|}$, for some $S \subseteq V$. Hence we studied some properties in this context. Let $p,q \in (0,1]$ be such that $p+q = 1$.

Proposition 3.7. *Let G = (V, E) be a graph. If $p = \frac{|N[S]|}{|V|}$, for some $S \subseteq V$,then $V - S$ is a q-dominating set in G.*

The i^{th} open neighbourhood of a vertex $v \in V(G)$, for $i \geq 0$ is defined by $N^i(v) = \{w \in V : d(v,w) = i\}$ and the i^{th} closed neighbourhood by $N^i[v] = \{w \in V : d(v,w) \leq i\}$.

Corollary 3.3. *Let G=(V, E) be a graph. If $p = \frac{|N^{k+1}[S]|}{|V|}$, where $k \geq 0$ for some $S \subseteq V$, then $V - N^k[S]$ is a q-dominating set in G.*

Proposition 3.8. *Suppose $p \in (0,1)$ and $p \neq \frac{|N[S]|}{|V|}$ for every $S \subseteq V$, then $\exists H,K \subseteq V$ such that $\frac{|N[H]|}{|V|} < p < \frac{|N[K]|}{|V|}$. Moreover, K is a p-dominating set and V-H is a q-dominating set.*

We investigated the behaviour of partial dominating sets in the case of graph complements too.

Proposition 3.9. *If $p = \frac{|N[S]|}{|V|}$ for some $S \subseteq V$, then S is a q-dominating set in G^c.*

Corollary 3.4. *Let G = (V, E) be a graph. If $p = \frac{|N^k[S]|}{|V|}$, where $k \geq 1$ for some $S \subseteq V$,then $N^{k-1}[S]$ is a q-dominating set in G^c.*

Theorem 3.2. *Suppose G is a n-vertex graph with maximum degree $\Delta(G)$ and $\gamma(G) = \frac{n}{\Delta(G)+1}$. Then $\gamma_p(G) = \lceil p\gamma(G) \rceil$ for all values of p.*

3.3 Properties related to eccentricity of a p-dominating set

For $v \in V$ and $S \subseteq V$, $d(v,S) = min\{d(v,s) : s \in S\}$ and $ecc(S) = max\{d(v,S) : v \in V\}$. We extended a partial dominating set to a dominating set using its eccentricity.

Proposition 3.10. *Let G be a connected graph. Suppose $p \in (0,1)$ and $S \subseteq V$ is a p-dominating set. Then ecc(S)=k iff $N^{k-1}[S], k \geq 2$ is a dominating set in G.*

Corollary 3.5. *Let G be a connected graph. Suppose $p \in (0,1)$ and $S \subseteq V$ is a p-dominating set. Then ecc(S)=k iff $N^i[S], i = 1,2,\ldots,k-2$ where $k \geq 2$ is not a dominating set and is a $\frac{|N^{i+1}[S]|}{|V|}$ dominating set in G.*

Proposition 3.11. *Suppose $p \in (0,1)$ and $S \subseteq V$ is a p-dominating set with ecc(S)=k where $k \geq 2$. Let V_i denote the set of vertices at a distance of i from S, where $i \geq 2$. Then $S \cup V_2 \cup V_4 \cup \ldots \cup V_k$ if k is even and $S \cup V_2 \cup V_4 \cup \ldots \cup V_{k-1}$ if k is odd are dominating sets.*

For any graph G, $diam(G) = max\{ecc(v) : v \in V\}$.

Proposition 3.12. *Let H be an induced subgraph of G with diam(H)=2. Then for* $p \in \left(0, \frac{|V(H)|}{|V(G)|}\right]$, $\gamma_p(G) \leq \delta(H)$.

Proposition 3.13. *Let H be an induced subgraph of G with* $diam(H) \geq 3$. *Then for* $p \in \left(0, \frac{|V(H)|}{|V(G)|}\right]$, $\gamma_p(\overline{G}) \leq 2$.

3.4 Properties in the line of classical domination

In [1], we come across the following theorem:

Theorem 3.3. *If G is a graph with no isolated vertices, then the complement V-S of every minimal dominating set S is a dominating set.*

We give a similar result in the context of partial domination.

Proposition 3.14. *Suppose G is a n-vertex graph having no isolated vertices. Then for any minimal p-dominating set S,* $N(S)$ *is also a p-dominating set.*

We also proved the following.

Proposition 3.15. *Every minimal p-dominating set can be extended to form a minimal dominating set.*

Proposition 3.16. *If G is a graph with no isolated vertices, then the complement V-S of every minimal p-dominating set S is a dominating set.*

Any acyclic graph is a forest. A subgraph whose vertex set is the same as that of G is a spanning subgraph of G. A spanning forest F of G is called maximum if it has the largest possible number of pendant edges among all spanning forests of G, this number being denoted by $\varepsilon_F(G)$. A spanning forest F is called a 2-forest if the distance between any two vertices in a connected component of F does not exceed two. In 1974 Nieminen proved one of the earliest results about the domination number.

Theorem 3.4. [*10*] *For any graph G,* $\gamma(G) + \varepsilon_F(G) = n$, *where* $\varepsilon_F(G)$ *denote the maximum number of pendant edges in a spanning forest of G.*

He used a method to construct $F(G)$, a spanning 2-forest of G. By modifying that method slightly, we constructed $F_p(S)$, a spanning 2-forest with respect to a p-dominating set S. This helped us in characterising a p-dominating set.

Proposition 3.17. *A p-dominating set S is a minimum p-dominating set iff the spanning forest* $F_p(S)$ *is a maximum 2-forest of <N[S]>.*

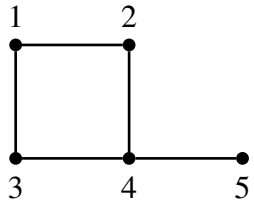
We also characterized a minimal p-dominating set in the way Ore[1] had characterized a minimal dominating set.

Theorem 3.5. *Let $S \subseteq V$ be a p-dominating set. Then it is a minimal p-dominating set iff each $u \in S$ satisfies one of the following two conditions.*
(i)$N(u) \subseteq N(S)$
(ii)there exists a vertex $v \in N(S)$ for which $N(v) \cap S = \{u\}$.

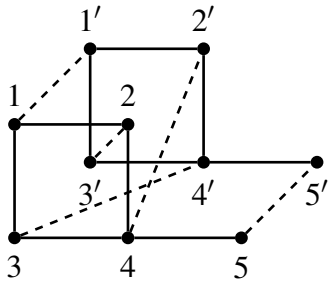
4 Partial Domination in Prisms of Graphs

In Chapter 4 we investigate the values of partial domination number for the prisms of graphs.

Let π be any permutation on $V(G)$. The prism πG of G with respect to π is obtained by taking two disjoint copies of G and joining each vertex v in one copy of G with $\pi(v)$ in the other copy by means of an edge. Figure 2b illustrates this.



(a) G



(b) Prism of G under $\pi = (2,3,4)$

Figure 2: Graph and Prism of Graph

Chartrand and Harary[11] in 1967 introduced the concept of prisms of graphs and they referred to these graphs as 'permutation graphs'. Except for the same coinage, these graphs have nothing in common with the permutation graphs[12] which were later defined in 1972. B. Piazza and R. Ringeisen[13] in 1991 named these graphs as generalized prism over G with respect to a permutation. One should not confuse these graphs with the prism graphs. Prism graphs are actually a particular case of these graphs when $\pi = 1$ and $G = C_n$.

The study of domination in prisms of graphs started in 2004 [14]. Since then many works[15, 16] related to various domination parameters were studied. These works and these graphs' potential for application motivated us to find partial domination numbers for prisms of various graphs.

4.1 Relations between p-domination number of a graph and that of its prisms

We found some relations between partial domination number of a graph and that of its prism.

Proposition 4.1. *Let G be any n-vertex graph and $q \in (0,1]$. Then for any permutation π of V(G) and for any $p \in (0, \frac{nq+\gamma_q(G)}{2n}]$, $\gamma_p(\pi G) \leq \gamma_q(G)$.*

We proved that the above bound is sharp in the case of paths and cycles.

Theorem 4.1. *Let G be P_n or C_n for $n \geq 3$. Then for any permutation π on V(G) and $p \in (0,1]$, $\gamma_{\frac{np+\gamma_p(G)}{2n}}(\pi G) = \gamma_p(G)$.*

Here paths and cycles on n-vertices are denoted by P_n and C_n respectively. As a corollary to the above result, we proved that with the γ-set of paths or cycles, one can dominate at least $\frac{2}{3}$rd of the vertices in their prisms.

Corollary 4.1. *Suppose G is P_n or C_n for $n \geq 3$ and $p \in (0,1]$, then $\gamma_{\frac{2}{3}p}(\pi G) \leq \gamma_p(G)$ and $\gamma_{\frac{2}{3}}(\pi G) \leq \lceil \frac{n}{3} \rceil$ for any permutation π on $V(G)$.*

The following two results characterize graphs G, for which $\gamma_{\frac{np+\gamma_p(G)}{2n}}(\pi G)$-values for a given $p \in (0,1]$ are either 1 or 2.

Remark 4.1. *For any graph G and $p \in (0,1]$, $\gamma_{\frac{np+\gamma_p(G)}{2n}}(\pi G) = 1$ iff $\gamma_p(G) = 1$.*

Proposition 4.2. *Let G be any graph with u_Δ as a vertex having the maximum degree $\Delta(G)$ and u'_Δ its mirror image in the second copy of G in πG. Then for $p \in (0,1]$, $\gamma_{\frac{np+\gamma_p(G)}{2n}}(\pi G) = 2$ iff one of the following two conditions holds for G:*
(i) $\gamma_p(G) = 2$
(ii)$\gamma_p(G) \geq 3$ and $\Delta(G) \geq \frac{np+\gamma_p(G)-4+i}{2}$ where $|N[u_\Delta] \cap N[u'_\Delta]| = i$ for $0 \leq i \leq 2$.

Corollary 4.2. *If G is a connected graph having atleast 2 vertices and having no P_5, C_5 or $K_n, n > 3$, then $p \in (0,1]$, $\gamma_{\frac{np+\gamma_p(G)}{2n}}(\pi G) = \gamma_p(G)$.*

A chain of inequalities involving the domination numbers of graphs G and their prisms πG, in the form of $\gamma(G) \leq \gamma(\pi G) \leq 2\gamma(G)$ has already been established in [17]. The following result generalizes this inequality.

Theorem 4.2. *Suppose $p \in (0,1]$ and π is any permutation on V(G), where G is any n- vertex graph. Then $\gamma_p(G) \leq \gamma_p(\pi G) \leq 2\gamma_p(G)$.*

If the proportion 'p' of the vertices that needs to be dominated in the prisms of graph G involve the maximum degree of G, then we calculated the corresponding γ_p-values for some classes of graphs G. We gave some methods and an algorithm to find those γ_p-sets.

Proposition 4.3. *Let G be a graph having an independent set $M = \{v_1, v_2, ..., v_k\}$ of k-vertices $(k \geq 1)$ each having the maximum degree $\Delta(G)$. If $N(v_i) \cap N(v_j) = \phi \forall v_i, v_j \in M$ then for $i = 1, 2, ..., k$, $\gamma_p(\pi G) = i$ for $p \in \left(\frac{(i-1)(\Delta(G)+2)}{2n}, \frac{i(\Delta(G)+2)}{2n} \right]$ for any permutation π on V(G).*

Proposition 4.4. *Let G be a n-vertex graph having a set of k-mutually non-adjacent vertices*$(k \geq 1)$ *say* $M = \{v_1, v_2, ..., v_k\}$ *each having the maximum degree* $\Delta(G)$. *Let G and* G' *be the two copies of G in* πG *for any permutation* π *on V(G). Let* $M' = \{v'_1, v'_2, ..., v'_k\}$ *be the copy of M in* G'. *If for each* $v_r \in M$, *there exists exactly one* $v_s \in M$ *such that* $|N(v_r) \cap N(v_s)| = 1$, *then for* $i = 1, 2, ..., k$, $\gamma_p(\pi G) = i$ *for* $p \in \left(\frac{(i-1)(\Delta(G)+2)}{2n}, \frac{i(\Delta(G)+2)}{2n}\right]$ *for the following permutations* π *on V(G):*

1. π=*1*
2. $\pi(v_r) = v'_s$ *and* $\pi(v_s) = v'_r$ *for each* v_r *and* v_s *as defined above.*
3. $\pi \neq 1$ *and* $\pi(v_r) \neq v'_s$ *or* $\pi(v_s) \neq v'_r$ *or both and* $\pi(v_i) \in M'$ $\forall v_i \in M$.

4.2 Universal γ_p-fixers and universal γ_p-doublers

As discussed earlier, for any graph G and for any permutation π on $V(G)$, $\gamma(G) \leq \gamma(\pi G) \leq 2\gamma(G)$. It has been defined in [14] that, G is called universal γ-fixer if $\gamma(G) = \gamma(\pi G)$ for all permutations π of V(G) and in [18] it was established that edgeless graphs are the only universal fixers. Burger and Mynhardt[14] called G to be a universal doubler if $\gamma(\pi G) = 2\gamma(G)$ for all permutations π of V(G). They characterized graphs which are universal doublers. Motivated by their works and in view of Theorem 4.2 we defined the following in the context of partial domination.

Definition 4.1. *Let* $p \in [0, 1]$. *G is called universal* γ_p*-fixer, if* $\gamma_p(G) = \gamma_p(\pi G)$ *for all permutations* π *of V(G) and G is called universal* γ_p*-doubler if* $\gamma_p(\pi G) = 2\gamma_p(G)$ *for all permutations* π *of V(G).*

For graphs G with $\gamma(G)$=1, we found values of 'p' for which G is a universal γ_p-fixer and G is a universal γ_p-doubler.

Proposition 4.5. *Let G be any n-vertex*$(n \geq 2)$ *graph with* $\gamma(G)$= *1. Then for any permutation* π *on V(G) and for any* $p \in (0, 1]$,

$$\gamma_p(\pi G) = \begin{cases} 1, & \text{for } p \in (0, \frac{n+1}{2n}] \\ 2, & \text{for } p \in (\frac{n+1}{2n}, 1] \end{cases}$$

Remark 4.2. *Suppose G is a graph with atleast 2 vertices and* $\gamma(G)$=*1. Then G is a universal* γ_p*-fixer for* $p \in (0, \frac{n+1}{2n}]$ *and is a universal* γ_p*-doubler for* $p \in (\frac{n+1}{2n}, 1]$.

As a particular case, when $p = \frac{1}{2}$ we found that G is a universal $\gamma_{\frac{1}{2}}$ - fixer.

Corollary 4.3. *Every graph having its domination number equal to 1 is a universal* $\gamma_{\frac{1}{2}}$ *- fixer as* $\frac{n+1}{2n} > \frac{1}{2}$.

Prisms of graphs play a role in structuring interconnection networks as mentioned in [17], [19] and dominating sets play a role in resource allocation problems in such networks. In any case, minimizing the domination number and achieving the optimum situation with the given resources is the basis behind partial domination. Here we focussed on $p = \frac{1}{2}$ and observed the effect of $\gamma_{\frac{1}{2}}$-set of G on $\gamma_{\frac{1}{2}}$-set of πG. We characterized complete bipartite graphs which are universal $\gamma_{\frac{1}{2}}$-fixers and which are universal $\gamma_{\frac{1}{2}}$- doublers.

Theorem 4.3. *Every complete bipartite graph $K_{m,n}, m \leq n$ is a universal $\gamma_{\frac{1}{2}}$ - fixer iff $m \leq 2$.*

Theorem 4.4. *For $m \leq n$, any complete bipartite graph $K_{m,n}$ is a universal $\gamma_{\frac{1}{2}}$ -doubler iff $m > 2$.*

We characterized cycles and paths which are universal $\gamma_{\frac{1}{2}}$-fixers and which are universal $\gamma_{\frac{1}{2}}$-doublers.

Theorem 4.5. *For $n \geq 1$, P_n is a universal $\gamma_{\frac{1}{2}}$ - fixer iff $n \in \{1,2,3,4,7,8\}$ and for $n \geq 3$, C_n is a universal $\gamma_{\frac{1}{2}}$ - fixer iff $n \in \{3,4,7,8\}$.*

Proposition 4.6. *P_n or C_n can be a universal $\gamma_{\frac{1}{2}} -$ doubler iff $n \in \{5,6\}$.*

5 Partial Domination Chain

In Chapter 5 we introduce the concepts of independent partial domination, partial independence, partial irredundance and establish a chain of inequalities involving them and other related parameters. We call this chain as 'partial domination chain'.

Suppose $G = (V,E)$ is a graph. A dominating set D of G is minimal if no proper subset of D is a dominating set. The minimum cardinality of a minimal dominating set is called the domination number of G denoted by $\gamma(G)$ and the maximum cardinality of a minimal dominating set is called the upper domination number denoted by $\Gamma(G)$. A set $I \subseteq V$ is called an independent set of vertices if no two vertices in I are adjacent to each other. The cardinality of a maximum independent set is called the vertex independence number of G and it is denoted by $\beta(G)$. The corresponding independent set is called the β-set of G. If a dominating set is independent too then it is called an independent dominating set and the minimum cardinality of an independent dominating set is called the independent domination number of G. We denote it by $i(G)$. Suppose $R \subseteq V$ and $u \in R$. For some $v \in V$, if $N[v] \cap S = \{u\}$, then v is a private neighbor of u. A set $R \subseteq V$ with each of its vertices having a private neighbor in V is an irredundant set of G. The minimum and maximum cardinalities of an irredundant set are denoted by $ir(G)$ and $IR(G)$ respectively.

The following well-known chain of inequalities that exist amongst domination, independent domination and irredundance parameters was first observed in 1978[20].

Theorem 5.1. *For any graph G, $ir(G) \leq \gamma(G) \leq i(G) \leq \beta(G) \leq \Gamma(G) \leq IR(G)$.*

This type of chain was observed in the case of few other domination parameters too. Also one of the open questions posed by B. Case et al., in their work[4] was to find out, what relationship, the above parameters have amongst themselves in the context of partial domination. Hence we tried to establish a similar kind of chain. We first defined independent partial domination.

5.1 Independent partial domination

Definition 5.1. *Suppose G=(V,E) is a simple graph and $p \in (0,1]$. A subset S of V is called an independent p-dominating set(IPD-set) if S is a p-dominating set and is independent. The minimum cardinality of an independent p-dominating set is called the independent p-domination number(IPD-number) and is denoted by $i_p(G)$.*

Following are some of the basic results.

Observations

1. For $p \in (0,1]$, $\gamma_p(G) \leq i_p(G)$
2. For any n-vertex graph G and for $p \in (0, \frac{\Delta+1}{n}]$, $i_p(G) = 1$.
3. For all $p \in (0,1]$, $i_p(G) = 1$ iff $i(G) = 1$.
4. For $p \in \left(\frac{n-1}{n}, 1\right]$, $i_p(G) = i(G)$.
5. For all $p \in (0,1]$, $i_p(G) \leq i(G)$.

We calculated exact values of independent partial domination numbers for paths, cycles and complete bipartite graphs.

Proposition 5.1. *Suppose P_n and C_n are paths and cycles respectively on n-vertices. Then for $n \geq 3$, $i_p(C_n) = i_p(P_n) = \lceil \frac{np}{3} \rceil$.*

Proposition 5.2. *For $m \leq n$, $i_p(K_{m,n}) = \begin{cases} 1, & \text{for } p \in (0, \frac{n+1}{m+n}] \\ i+1, & \text{for } p \in (\frac{n+i}{m+n}, \frac{n+(i+1)}{m+n}] \text{ where } 1 \leq i \leq m-1 \end{cases}$*

Also $i_p(K_{m,n}) \leq m$.

We found a bound for independent partial domination number in terms of independent domination number.

Theorem 5.2. *For any graph G with independent domination number i(G) and $p \in (0,1]$, $i_p(G) \leq \lceil p.i(G) \rceil$.*

For $p,q \in (0,1]$ with $p+q=1$, we found the following bound.

Proposition 5.3. *Let G be any graph with independent domination number i(G) and $p,q \in (0,1]$ with $p+q=1$. Then $i_p(G)+i_q(G) \leq i(G)+1$.*

We designed an algorithm for constructing minimal independent dominating set from a minimal independent partial dominating set.

Proposition 5.4. *Every minimal IPD-set can be extended to form a minimal independent dominating set.*

Allan and Laskar[21] in 1978, proved that if a graph is claw-free then it has the same domination and independent domination numbers. We proved an analogous result in the context of partial domination.

Proposition 5.5. *If a graph G is claw-free, then $\gamma_p(G) = i_p(G)$ for all $p \in (0,1]$.*

Corollary 5.1. *If L(G) is the line graph of a graph G, then $\gamma_p(L(G)) = i_p(L(G))$.*

5.2 Partial independence number

As a next step in our quest for the partial domination chain, we defined partial independence number.

Definition 5.2. *Suppose $G=(V,E)$ is a graph and $p \in (0,1]$. A set S of independent vertices is called a p-independent set in G if $N[S] \subseteq V(H)$ for some induced subgraph H of G with $|V(H)| \geq np$. A p-independent set S is said to be p-maximal if S is a maximal independent set in $V(H)$. A maximal p-independent set S is said to be min-max p-independent set if $T \subset S$ is not p-maximal. Partial independence number or p-independence number is the maximum cardinality of a min-max p-independent set and is denoted by $\beta_p(G)$ and the associated induced subgraph H is denoted by H_p.*

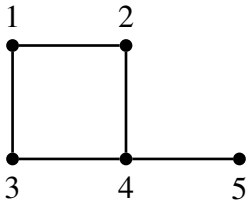


Figure 3: Graph with $\beta_{\frac{4}{5}} = 2$

For the graph in Figure 3, $\{2,5\}$ is a $\beta_{\frac{4}{5}}$-set. From the above definition, one can observe that for every maximal p-independent S, $N[S] = V(H)$ and thus S is a an independent p-dominating set. Also, the minimum cardinality of a maximal p-independent set is the independent p-domination number of G for any $p \in (0,1]$. Thus we had the following inequality.

Proposition 5.6. *For $p \in (0,1]$, $\gamma_p(G) \leq i_p(G) \leq \beta_p(G)$.*

From the following relation we found that for $p \in (0,1]$, p-independence number of a graph G is bounded by its independence number.

Proposition 5.7. *If $p_1 \leq p_2$, then $\beta_{p_1}(G) \leq \beta_{p_2}(G)$.*

For $p \in (0,1]$, B. Case et al. in [4] defined the upper p-domination number of a graph G to be the maximum cardinality of a minimal p-dominating set and denoted it by $\Gamma_p(G)$.

Proposition 5.8. *Every min-max p-independent set is a minimal p-dominating set.*

From the above result, we ended up with the following inequality.

Corollary 5.2. *For $p \in (0,1]$, $\beta_p(G) \leq \Gamma_p(G)$.*

From Proposition (5.6) and Corollary (5.2) we obtained the following chain of inequalities which is a part of our partial domination chain
For $p \in (0,1]$, $\gamma_p(G) \leq i_p(G) \leq \beta_p(G) \leq \Gamma_p(G)$. It is true that any proper subset of a $\beta-$ set is a p-independent set for some values of p. We gave a method for obtaining β_p-sets from a $\beta-$set of a graph and we also gave the exact values of p for which such sets exist.

Lemma 5.1. *Suppose S is a $\beta-$set of a graph G and $T \subset S$. Then T is a min-max $\frac{|N[T]|}{n}$-independent set.*

Theorem 5.3. *Let $\mathscr{B}_i$ denote the set of all i-element subsets of a β-set of a graph G for $1 \leq i \leq \beta(G)$. Let $B_i \in \mathscr{B}_i$ be such that $|N[B_i]| = min\{|N[X]|/X \in \mathscr{B}_i\}$. Then*
(i) B_i is a β_p-set for $p = \frac{|N[B_i]|}{n}$
(ii) For $0 < p \leq \frac{|N[B_1]|}{n}$, $\beta_p = 1$ and B_1 is a β_p-set.

5.3 Partial irredundance number

Next we defined partial or p-irredundance numbers for a graph G for $p \in (0,1]$.

Definition 5.3. *Suppose $G = (V,E)$ is a graph and $p \in (0,1]$. A set S of vertices is called a p-irredundant set in G if S is irredundant in an induced subgraph H of G with $N[S] \subseteq V(H)$ and $|V(H)| \geq np$. A p-irredundant set S is said to be p-maximal if S is a maximal irredundant set in H. A maximal p-irredundant set S is said to be min-max p-irredundant set if $T \subset S$ is not p-maximal. Partial irredundance number or p-irredundance number and upper p-irredundance number are the minimum and the maximum cardinalities of a min-max p-irredundant set respectively and are denoted by $ir_p(G)$ and $IR_p(G)$ with the associated induced subgraphs being denoted by H_p.*

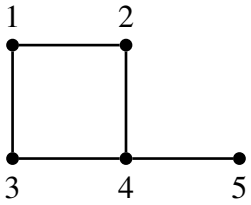


Figure 4: Graph with $ir_{\frac{4}{5}} = 1$ and $IR_{\frac{4}{5}} = 2$

For the graph in Figure 4, $\{4\}$ is an $ir_{\frac{4}{5}}$-set and $\{2,5\}$ is an $IR_{\frac{4}{5}}$-set.
We proved the following properties of p-irredundant sets which in turn led us towards our destination, the partial domination chain.

Proposition 5.9. *A p - dominating set is a minimal p - dominating set iff it is p - dominating and p - irredundant.*

Proposition 5.10. *Every minimal p - dominating set is a min-max p - irredundant set. Hence for $p \in (0,1]$, $ir_p(G) \leq \gamma_p(G)$ and $\Gamma_p(G) \leq IR_p(G)$.*

Proposition 5.11. *For $p \in (0,1]$, $ir_p(G) \leq \gamma_p(G) \leq i_p(G) \leq \beta_p(G) \leq \Gamma_p(G) \leq IR_p(G)$.*

For every graph G, both p-irredundance and upper p-irredundance numbers are bounded above by their irredundant counterparts in G.

Proposition 5.12. *For $p_1, p_2 \in (0,1]$, $ir_{p_1}(G) \leq ir_{p_2}(G)$ and $IR_{p_1}(G) \leq IR_{p_2}(G)$ if $p_1 \leq p_2$.*

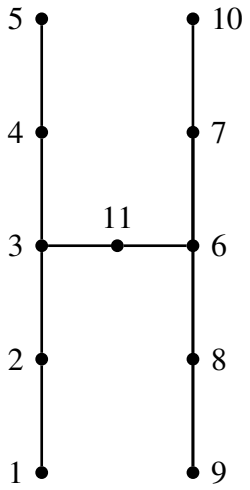
5.4 Relations between the parameters in the partial domination chain

We also found some more properties of p-irredundant sets which were similar to those of irredundant sets and also some relations that exist amongst the parameters that appear in the partial domination chain.

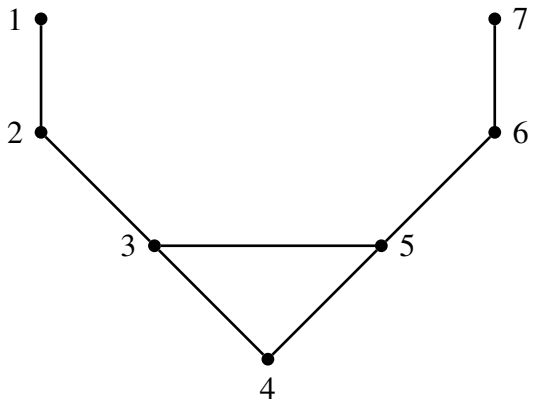
Proposition 5.13. *For $p \in (0,1]$, $\frac{\gamma_p(G)}{2} < ir_p(G) \leq \gamma_p(G) \leq 2ir_p(G) - 1$.*

Proposition 5.14. *If a graph G has no isolated vertices and $S \subseteq V$ is a p - irredundant set with H_p as its associated induced subgraph, then $V(H_p) - S$ is a p - dominating set in G and it dominates H_p.*

Proposition 5.15. *If a graph G has no isolated vertices and $S \subseteq V$ is a largest min-max p - irredundant set with H_p as its associated induced subgraph, then for $q = \frac{|V(H_p)|}{n}$, $\gamma_q(G) + IR_q(G) \leq nq$. Also for the above 'q',*
$\gamma_q(G) + \Gamma_q(G) \leq nq$
$\gamma_q(G) + \beta_q(G) \leq nq$



(a) H Graph



(b) A-L Graph

$ir_q(G) + \Gamma_q(G) \leq nq$
$ir_q(G) + IR_q(G) \leq nq$
$ir_q(G) + \beta_q(G) \leq nq$

We found some graphs for which some of the parameters are equal.

Proposition 5.16. *If G is a chordal graph containing no H or A−L graph as induced subgraph, then for* $p \in (0,1]$, $ir_p(G) = \gamma_p(G)$.

Proposition 5.17. *If T is a tree containing no induced H subgraph then for* $p \in (0,1]$, $ir_p(G) = \gamma_p(G)$.

Proposition 5.18. *If a graph G is* $K_{1,3}$ *or A-L graph free, then for* $p \in (0,1]$, $ir_p(G) = \gamma_p(G) = i_p(G)$.

A set $S \subseteq V$ is called a stable transversal if $|S \cap C| = 1$ for every maximal clique C of G. If G and each of its induced subgraphs have a stable transversal, then G is said to be strongly perfect.

Proposition 5.19. *For any strongly perfect graph and for* $p \in (0,1]$, $\beta_p(G) = \Gamma_p(G) = IR_p(G)$.

Proposition 5.20. *If G is a bipartite graph or a chordal graph, then for for* $p \in (0,1]$, $\beta_p(G) = \Gamma_p(G) = IR_p(G)$.

5.5 p-well covered graphs

Next we concentrated on a special class of graphs called well-covered graphs which was first studied by M.Plummer in 1970[22].

Definition 5.4. *A graph G is said to be well-covered if* $i(G) = \beta(G)$.

We give an analogous definition in the context of partial domination.

Definition 5.5. *For* $p \in (0,1]$*, a graph G is said to be p-well covered if* $i_p(G) = \beta_p(G)$*. A graph G is said to be universal p-well covered if* $i_p(G) = \beta_p(G)$ *for all* $p \in (0,1]$ *and it is said to be partial p-well covered if* $i_p(G) \neq \beta_p(G)$ *for atleast one* $p \in (0,1]$.

Here we observed the following

Remark 5.1. *Every graph G having no isolated vertices is p-well covered for* $p \in \left(0, \frac{\delta(G)+1}{n}\right]$ *and it is p-well covered for* $p \in \left(0, \frac{\Delta(G)+1}{n}\right]$ *if G is regular.*

Proposition 5.21. *If a graph G is universal p-well covered, then it is regular and well-covered.*

We characterized complete bipartite graphs with respect to p-well coveredness.

Proposition 5.22. *Every complete bipartite graph* $K_{m,n}$ *is partial p-well covered iff* $m < n$ *and is universal p-well covered iff* $m = n$.

Proposition 5.23. *Every path* $P_n, n \geq 3$ *is p-well covered if* $p \in \left(0, \frac{2}{n}\right] \cup \left(\frac{3}{n}, \frac{4}{n}\right]$ *and every cycle* $C_n, n \geq 6$ *is p-well covered if* $p \in \left(0, \frac{5}{n}\right]$.

Proposition 5.24. *Every path* $P_n, n \geq 3$ *and every cycle* $C_n, n \geq 6$ *is partial p-well covered.*

Proposition 5.25. *Every well-covered graph G is universal p-well covered if* $i(G) \leq 2$ *and G is regular.*

Corollary 5.3. *Suppose G is a connected, regular and well-covered graph having no induced* P_4 *or* C_4*. Then G is universal p-well covered.*

Theorem 5.4. *Suppose G is a well-covered graph with* $S \subseteq V(G)$ *be such that* $|S| = \beta(G) = i(G)$*. Let* $v \in V(G)$ *be such that* $S - N[v] \neq \phi$*. Then for* $p = \frac{|N[S-N[v]]|}{|V(G)|}$*, G is p-well covered.*

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LIST OF PUBLICATIONS

1. L. P. Nithya and J. V. Kureethara, “On Some Properties of Partial Dominating Sets", *AIP Conference Proceedings*, vol. 2236, no. 1, 2020. [Online].
   Available: https://aip.scitation.org/doi/abs/10.1063/5.0006808.

2. L. P. Nithya and J. V. Kureethara, “Partial Domination in Prisms of Graphs", manuscript accepted for publication in *Italian Journal of Pure and Applied Mathematics*.

3. L. P. Nithya and J. V. Kureethara, “Independent Partial Domination in Graphs" manuscript submitted to *CUBO, A Mathematical Journal*.

LIST OF CONFERENCE PRESENTATIONS

1. Paper titled “Some Properties of p-Dominating Vertex Sets in Graphs" was presented in the “II International Conference on Science, Engineering, Technology and Social Sciences" held at Kuriakose Elias College, Kottayam during January 18-19, 2018.

2. Paper titled “On Some Properties of Partial Dominating Sets" was presented in the “International Conference on Theoretical Computer Science and Discrete Mathematics" held at SSN College of Engineering, Chennai during December 3-5, 2018.

3. Paper titled “Properties of Partial Dominating Vertex Set of a Graph" was presented in the “National Conference on Recent Trends in Mathematics and its Applications" held at GITAM School of Technology, Bengaluru during December 21-22, 2018.

4. Paper titled “Partial Domination in Prisms of Graphs" was presented in the “International Conference on Emerging Trends in Graph Theory" held at CHRIST(Deemed to be University), Bengaluru during February 27-28, 2019.

5. Paper titled “Independent Partial Domination" was presented in the “National Conference on Discrete Mathematics and its Applications" held at CHRIST(Deemed to be University), Bengaluru on 15th December, 2019.